Asymptotische Statistik
Winter term 2018/19
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Exercise sheet 1

For the whole exercise sheet we consider a probability space $(\Omega, \mathcal{A}, \mathbb{P})$.

Exercise 1
Show that the following assertions hold.

(i) $o_{\mathbb{P}}(1) + o_{\mathbb{P}}(1) = o_{\mathbb{P}}(1)$

(ii) $o_{\mathbb{P}}(1) = O_{\mathbb{P}}(1)$

(iii) $o_{\mathbb{P}}\left(O_{\mathbb{P}}(1)\right) = o_{\mathbb{P}}(1)$

(iv) $O_{\mathbb{P}}(1) + O_{\mathbb{P}}(1) = O_{\mathbb{P}}(1)$

For (iv) and (v) let all considered random variables be $\mathbb{R}$-valued.

(v) $\left(1 + o_{\mathbb{P}}(1)\right)^{-1} = O_{\mathbb{P}}(1)$

(vi) $O_{\mathbb{P}}(1) \cdot o_{\mathbb{P}}(1) = o_{\mathbb{P}}(1)$

(6 points)

Exercise 2
For $n \in \mathbb{N}$ let X_n be $\mathbb{R}$-valued random variables and $X_n \xrightarrow{d} X$ for an additional random variable X. Show that $X_n = O_{\mathbb{P}}(1)$.

(2 points)

Exercise 3
Let $k, r \in \mathbb{N}$ and consider a map $g : \mathbb{R}^k \to \mathbb{R}$ with $g(0) = 0$. For $n \in \mathbb{N}$ let X_n be $\mathbb{R}^k$-valued random variables with $X_n = o_{\mathbb{P}}(1)$. Show the following statements:

(i) If $g(x) = o(\|x\|^r)$ for $x \to 0$ then $g(X_n) = o_{\mathbb{P}}(\|X_n\|^r)$.

(ii) If $g(x) = O(\|x\|^r)$ for $x \to 0$ then $g(X_n) = O_{\mathbb{P}}(\|X_n\|^r)$.

Recall that we say $g(x) = o(||x||^r)$ for $x \to 0$ if $\frac{g(x)}{||x||^r} \to 0$ for $x \to 0$ and that $g(x) = O(||x||^r)$ for $x \to 0$ if there exists $\varepsilon > 0$ with $\sup_{\|x\|<\epsilon} \frac{g(x)}{\|x\|^r} < \infty$.

(2 points)

Exercise 4
For $n \in \mathbb{N}$ let X_n be $\mathbb{R}$-valued random variables and A_n a sequence of events, i.e. sets in $\mathcal{A}$. Assume that

$$X_n = o_{\mathbb{P}}(1) \text{ on } A_n,$$

meaning that $X_n \mathbb{1}_{A_n} = o_{\mathbb{P}}(1)$.
Show that $X_n = o_{\mathbb{P}}(1)$ if $\mathbb{P}(A_n) \to 1$ for $n \to \infty$.

(2 points)

The solutions are to be handed in at the designated **box number 03** on the first floor of INF 205 (Mathematikon) in front of the Dekanat by **Tuesday, October 23rd 2018** before 9 c.t..