UNIVERSITÄT HEIDELBERG
ZUKUNFT
SEIT 1386

Exercise sheet 4

Exercise 1

Let $\Theta \subseteq \mathbb{R}^d$ and let $Q_n : \Theta \to \mathbb{R}$ be random functions for $n \in \mathbb{N}$ and $Q : \Theta \to \mathbb{R}$ a deterministic function with unique minimizer θ_0. Now assume that

(i) Q and Q_n are convex functions for all $n \in \mathbb{N}$,

(ii) $Q_n(\theta) \xrightarrow{p} Q(\theta)$ for all $\theta \in \Theta$.

Show that $\hat{\theta}_n := \arg\min_{\theta\in\Theta} Q_n(\theta)$ is a consistent estimator for θ_0. **(3 points)**

Exercise 2

We continue Exercise 4 from Exercise sheet 2 and Exercise 1 from Exercise sheet 3 and therewith all notation and assumptions made. Construct the Wald test for the hypothesis $\theta_{0,2} = 0$. **(3 points)**

Exercise 3

Consider the $\mathbb{R}^d$-valued random variable $X \sim \mathcal{N}_d(0, V)$. Choose the matrix V^- such that $VV^-V = V$ and show that $X^T V^- X \sim \chi_2(k)$ where k is the rank of V. **(4 points)**

Exercise 4

Consider a sequence $(Z_i)_{i=1,...n}$ of *iid* random variables with $Z_1 \sim P_n(\theta_0)$ where P_n is is a distribution that depends on n. Recall the usual notation from the lecture and define

$$\hat{\theta}_n := \arg\min_{\theta\in\Theta} Q_n(\theta) = \arg\min_{\theta\in\Theta} \frac{1}{n}\sum_{i=1}^{n} q(Z_i, \theta).$$

For some $\theta_0 \in \text{int}\Theta$ and some $\delta \in \mathbb{R}^d$ assume that

(a) $\hat{\theta}_n = \theta_0 + o_{\mathbb{P}}(1)$,

(b) $\sqrt{n}(\hat{\theta}_n - \theta_n) \xrightarrow{d} \mathcal{N}(0, H_0^{-1}\Omega_0 H_0^{-1})$ with $\theta_n = \theta_0 + n^{-\frac{1}{2}}\delta$.

Recall the Wald test statistic W_n for the hypothesis $A(\theta_0) = 0$ where $A : \Theta \to \mathbb{R}^p$ is a continuously differentiable function. Determine the asymptotic distribution of W_n and the rejection probability of the Wald test under P_n.

(4 points)

The solutions are to be handed in at the designated **box number 03** on the first floor of INF 205 (Mathematikon) in front of the Dekanat by **Tuesday, November 13th 2018** before 9 c.t..