Asymptotische Statistik
Winter term 2018/19
Lecturer: Prof. Dr. Enno Mammen
Assistant: Marilena Müller

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Exercise sheet 5

Exercise 1

Consider a sequence of n iid $\mathcal{N}(\mu_0, \sigma_0^2)$ random variables with $\mu_0 \neq 0$. Construct the Wald test for the hypothesis $\frac{\sigma_0}{\mu_0} = 1$. **(4 points)**

Exercise 2

Let $F_n : \mathbb{R}^d \to \mathbb{R}$ be a random functions. Assume that for every $M \in \mathbb{R}$ we have $\sup_{\|x\| \leq M} F_n(x) = o_{\mathbb{P}}(1)$. Show that there exists a sequence $M_n \to \infty$ for $n \to \infty$ such that

$$\sup_{\|x\| \leq M_n} F_n(x) = o_{\mathbb{P}}(1).$$

(4 points)

Exercise 3

Prove Exercise 1 from the lecture. Recall the notation from chapter 6 of the lecture.
Show that there exist c', c^* and c^{**} depending only on ν_1 and ν_2 such that

$$\mathbb{P}(\|\hat{m}_n - m_0\|_n > \delta) \leq c^* \exp(-c^{**} n \delta^2)$$

for $\delta \geq c' \sqrt{\frac{p}{n}}$. **(2 points)**

Exercise 4

Prove Lemma 6.1.5 from the lecture. **(4 points)**

Exercise 5

Do Exercise 3 from the lecture, i.e. prove an exponential bound for $\|\hat{m}_n^* - m_0\|_n$ for the respective $\hat{m}_n^*$ by showing an exponential bound for $\|\epsilon\|_n$. **(2 points)**

The solutions are to be handed in at the designated **box number 03** on the first floor of INF 205 (Mathematikon) in front of the Dekanat by **Tuesday, November 20th 2018** before 9 c.t..