



Exercise sheet 6

Exercise 1

Prove Exercise 4 from the lecture.

State and prove an exponential inequality for $\|\hat{m}_n - m_0\|_n$ for the class of functions

$$\mathcal{M}_n = \left\{ m : [0, 1] \rightarrow \mathbb{R} : \int |m^{(k)}(z)| dz \leq C, \|m\|_n \leq L \right\}$$

with $C, M > 0$ and $k \geq 2$.

(4 points)

Exercise 2

Prove Theorem 6.2.1 from the lecture.

(6 points)

Exercise 3

Do Exercises 6 and 7 from the lecture.

(a) Finish the proof of Theorem 6.3.1 from the lecture, i.e. show that

- $2\langle \hat{m}_n - m_0, \varepsilon \rangle_n > \lambda_n^2 I^\nu(m_0)$,
- $I^\nu(\hat{m}_n) > 1$

imply that $\|\hat{m}_n - m_0\|_n = O_{\mathbb{P}}(\lambda_n)$.

(b) Show that $I(\hat{m}_n) = O_{\mathbb{P}}(1)$.

(4+2 points)