Asymptotische Statistik
Winter term 2018/19
Lecturer: Prof. Dr. Enno Mammen
Assistant: Marilena Müller

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Exercise sheet 9

Exercise 1

Show that the Orlicz norm as introduced in Definition 9.2.2 indeed is a pseudo norm in the following sense.
Let $\mathbb{X}$ be the space of integrable $\mathbb{R}$-valued random variables for which the Orlicz norm w.r.t. ψ is finite where $\psi : [0, \infty) \to [0, \infty)$ is a convex, increasing and unbounded function with $\psi(0) = 0$. Show that it holds for all $X, Y \in \mathbb{X}$ that

(i) $\|X\|_\psi = 0 \Leftrightarrow X = 0$ a.s.,

(ii) $\|\alpha X\| = |\alpha| \|X\|_\psi$ for all $\alpha \in \mathbb{R}$,

(iii) $\|X + Y\|_\psi \leq \|X\|_\psi + \|Y\|_\psi$. **(3 points)**

Exercise 2

Consider $\psi_p : [0, \infty) \to [0, \infty), x \mapsto \exp(x^p) - 1$ and let X be an integrable random variable. Show that the function

$$p \mapsto (\log(2))^{\frac{1}{p}} \|X\|_{\psi_p}$$

is nondecreasing for $p \in [1, \infty)$.
Note, that the function $\phi : [0, \infty) \to [0, \infty)$ with

$$\psi_p\big(\log(2)^{\frac{1}{p}} x\big) = \phi\Big(\psi_q\big(\log(2)^{\frac{1}{q}} x\big)\Big)$$

for all $x \in [0, \infty)$ is concave and satisfies $\phi(1) = 1$ for $q \geq p$. **(4 points)**

Exercise 3

Do Exercise 14, i.e. prove Lemma 9.4.9:
Let $(\mathcal{X}, \mathcal{A}, \mathbb{P})$ be a probability space. For classes $\mathcal{D}, \mathcal{D}_1, \mathcal{D}_2$ of measurable subsets of $\mathcal{X}$ it holds that

(i) $m_{\mathcal{D}}(n) = m_{\{\mathcal{X} \setminus D : D \in \mathcal{D}\}}(n)$,

(ii) $m_{\mathcal{D}_1 * \mathcal{D}_2}(n) \leq m_{\mathcal{D}_1} \cdot m_{\mathcal{D}_2}(n)$

where $\mathcal{D}_1 * \mathcal{D}_2$ denotes $\{D_1 * D_2 : D_1 \in \mathcal{D}_1, D_2 \in \mathcal{D}_2\}$ with $* \in \{\cap, \cup, \setminus\}$.

(3 points)

Exercise 4

Do Exercise 15, i.e. prove (i) of Lemma 9.4.11:
For a finite dimensional vector space $\mathcal{F}$ of functions it holds that

$$V(\text{sgr}(\mathcal{F})) \leq \dim(\mathcal{F}) + 2.$$

(6 points)

The solutions are to be handed in at the designated **box number 03** on the first floor of INF 205 (Mathematikon) in front of the Dekanat by **Tuesday, December 18th 2018** before 9 c.t..